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Using Game Theory to Explore the Multinational Supply Chain Production Inventory Models of Various Carbon Emission Policy Combinations

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Abstract: This study uses Stackelberg game theory, considering different combinations of carbon emission reduction policies and that high-carbon-emission enterprises may face various carbon emission reduction regulations, to explore the production inventory problems in a multinational supply chain system. The purpose is to determine the manufacturer's optimal production, shipping, carbon reduction investment, and the retailer's replenishment under the equilibrium for different carbon emission policy combinations. To develop the production inventory models, this study first develops the total profit and carbon emission functions of the supply chain members, respectively, and then obtains the optimal solutions and total profits of the manufacturer and the retailer under different carbon emission policy combinations through the mathematical analysis method. Further, this study used several numerical examples to solve and compare the proposed models. The results of numerical analysis show that regardless of the increase in carbon price or carbon tax, the manufacturer and retailer will adjust their decisions to reduce carbon emissions. Specifically, an increase in the carbon price contributes to an increase in the total profit of manufacturers, while an increase in the carbon tax reduces the total profit of manufacturers. This study also explores a sensitivity analysis on the main parameters and has yielded meaningful management insights. For instance, in cases where low-carbonization strategies are required, the manufacturer or retailer can effectively reduce the carbon emissions resulting from production or purchasing activities, thereby significantly reducing overall carbon emissions. It is believed that the results of this study can provide enterprises/supply chains with reference to their respective production, transportation, carbon reduction investment, and inventory decisions under carbon emission policies, as well as information on partner selection and how to adjust decisions under environmental changes.

Keywords: inventory; game theory; carbon emissions; multinational supply chain

MSC: 90B05

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1. Introduction

With the rise of Environment, Social, and Governance (ESG) issues and the rise of sustainability awareness, the goal of carbon neutrality has begun to be taken seriously, and various countries have developed their own carbon emission policies. ESG issues have attracted the attention of the government, enterprises, and academia since they appeared in the United Nations report in 2006, and have been widely used as an important indicator to measure the sustainability and social influence of enterprises. In 2015, the United Nations General Assembly further signed the Global Sustainability Development Goals (SDGs), which cover the environmental, economic, and social aspects, and are also the goals of continuous global efforts. In the above issues and sustainable development goals, environmental protection is regarded as one of the critical issues. According to The Global Risks Report 2023 [1], four of the top five risks in the world are related to global warming and extreme climate. All countries agree that carbon dioxide emissions are the main driving factor of global climate change, and its threat is increasing year by year. Greenhouse gas emissions (mainly carbon dioxide) lead to global warming [2], and then cause extreme weather such as heat waves, droughts, forest fires, rainstorms, floods and so on, which not only affect everyone's life, but also bring serious losses of life and property. Therefore, green thinking such as environmental protection, carbon dioxide emission reduction, and alternative energy is no longer a slogan, and concepts such as green policy, green energy, and green supply chain have emerged one after another. In the supply chain system of the manufacturing industry, carbon emissions from sourcing, production, inventory, transportation, sales, and use of goods are important factors causing extreme climate. Since the Kyoto Protocol was signed by 84 countries in 2005 to mitigate carbon emissions, the governments of member countries have begun to actively promote some measures to limit greenhouse gas emissions [3]. Such developments include the creation of new alternative energy or renewable energy, the formulation of energy conservation and carbon reduction regulations, the advancement of carbon trading markets like the EU Emissions Exchange, the Chicago Climate Exchange, the Tianjin Emissions Exchange, and the implementation of carbon taxes. In 2015, the 21st Conference of the Parties to the United Nations Framework Convention on Climate Change in Brazil, France, also adopted a historic agreement, which became a new legally binding agreement on greenhouse gas reduction after the Kyoto Protocol. It can be seen that the control of greenhouse gas emissions is a new trend, and this control measure will inevitably affect the operation strategy of enterprises. With the rise of environmental awareness, in the face of multiple pressures from the government, customers, and other stakeholders, enterprises must also try to reduce the environmental and social impacts of their operations by reducing carbon emissions in addition to pursuing profits [4].

When facing the issue of carbon emission reduction, most enterprises focus on reducing carbon emissions through some physical processes, such as replacing equipment or facilities with lower energy efficiency, redesigning product packaging and regulation, or using low-pollution energy [5]. However, some scholars have found that enterprises can significantly reduce carbon emissions without significantly increasing costs through operational management, such as inventory management [6]. Therefore, many issues about carbon emission or carbon footprint management in inventory management have been discussed gradually. At first, some scholars incorporated carbon emission reduction policies, including restrictive carbon emissions, carbon taxes, carbon cap-and-trade, or carbon offsets into the economic order quantity (EOQ) or economic production quantity (EPQ) models, such as [7–11]. Wu et al. [12], Shen et al. [13], Rout et al. [14], Qi et al. [15], and Huang et al. [16] discussed the production inventory model including manufacturers and retailers from the perspective of supply chain integration. Astanti et al. [17] developed a vendor managed inventory model to analyze the correlation between carbon price and total carbon emissions. However, the above research literature assumes that supply chain members face the same carbon emission reduction policy. In practice, supply chain members may face different carbon emission reduction policies, especially considering a transnational supply chain from different countries. Cheng et al. [18] explored the combination of pre-sale and credit transactions, which affects the formulation of the best pricing and inventory model under different carbon tax policies, while Lu et al. [19] explored and compared supply chain production inventory issues for different combinations of carbon emission reduction policies. Shen et al. [20] proposed a supply chain competitive inventory model that considers both carbon trading and carbon tax as a mixed carbon emission reduction policy, which is suitable for industries with high carbon emissions.

In addition, when the managers of enterprises are engaged in decision making (including production, transportation, inventory, etc.), the situations they face are not independent, and may be affected by the choices of other decision-makers, that is, the decision-making situations are interactive. Therefore, how to explore the interaction between members about inventory and investment-related decisions in the supply chain system, to minimize the total inventory-related cost or maximize the total profit of each member, is also a topic worthy of discussion. Academically, many scholars have used the concept of game theory to explore the interactive decision making of inventory between manufacturers and retailers, and established a series of supply chain competitive inventory models. Most of them discuss the optimal decisions of buyers and sellers from the perspective of Stackelberg game theory, that is, considering one member of the supply chain as the leader and the other as the follower [21–28]. However, as far as we know, there is no literature using the concept of game theory to discuss the dependent inventory decision problem of supply chain members under different carbon emission policy combinations or mixed carbon emission policies.

Therefore, from the perspective of game theory, this study explores the production inventory of retailers and manufacturers in the supply chain under different carbon emission reduction policy combinations and mixed carbon emission reduction policies. In this study, the total profit and carbon emission functions of supply chain members are established, respectively. Then, we consider two carbon emission reduction policy combinations of carbon tax and carbon cap-and-trade. (1) The retailer faces carbon tax policy, and the manufacturer faces a mixed policy of carbon cap-and-trade and carbon tax at the same time; (2) the retailer faces a carbon cap-and-trade policy, and the manufacturer faces a mixed policy of carbon cap-and-trade and carbon tax at the same time. Exchange rate issues are also taken into account in the model. The problems to be solved in this study are as follows. First, how does the retailer determine the quantity of replenishment and the length of the cycle, and how does the manufacturer determine the quantity and frequency of delivery in the case that the members of the supply chain system make decisions that affect each other and have different carbon emission policies. Second, the sensitivity analysis of the optimal equilibrium decision, individual profit, and carbon emission quantity of the manufacturer and retailer is carried out by comparing different carbon emission reduction portfolio scenarios. Finally, the influence of the model parameters on the optimal equilibrium decision is discussed. In the aspect of model development, firstly, the profit and carbon emission functions of supply chain members are established. Then, under different carbon emission policy combinations, the Stackelberg game of a single leader and a single follower is considered, and the solution method of finding the optimal equilibrium solution value of the manufacturer and the retailer is developed for two different combination scenarios. Furthermore, this study illustrates the solution procedure through several more reasonable numerical examples and conducts sensitivity analysis on the main model numbers. It is hoped that some meaningful management implications can be obtained to provide a reference for enterprises to make relevant decisions.

2. Literature Review

The literature review of this study is mainly divided into two parts, including the consideration of carbon emission reduction policies and the use of game theory to explore the inventory problem, which are described as follows.

2.1. Inventory Model Considering Carbon Emission Reduction Policies

At the earliest, Hua et al. [7] developed an economic ordering quantity (EOQ) model taking into account the carbon cap-and-trade policy. Arslan and Turkay [8] also modified the traditional EOQ model based on carbon emission policies such as carbon tax, carbon cap-and-trade, and carbon offset. Chen et al. [5] further developed conditions that can reduce carbon emissions by adjusting order quantities. Since then, many scholars have considered various practical issues and explored how to optimize production and inventory strategies in company or supply chain management under various carbon emission reduction policy frameworks. Zhang and Xu [28] further explored the multi-item newsboy model under the carbon cap-and-trade and limited warehouse capacity. Battini et al. [10] incorporated the concept of carbon footprint management, established a Sustainable Economic Order Quantity (S-EOQ) model, and compared it with the traditional EOQ model. He et al. [9] addressed the issues of production lot sizing under cap-and-trade and carbon tax regulations based on the EOQ model. Hovelaque and Bironneau [11] followed Hua et al. [7] and developed a sustainable EOQ model in which demand is dependent on selling price and carbon emissions. Dye and Yang [29] developed a deteriorating inventory model with trade credit under carbon cap-and-trade and carbon offset regulations. Hua et al. [30] also established a sustainable inventory model for deteriorating items and considered the freshness-dependent demand. Recently, Cheng et al. [18] proposed a comprehensive inventory model including price-dependent demand, pre-sale incentives, advance sales, trade credit, and carbon tax policies based on the EOQ model. On the other hand, Ghosh et al. [31] first developed a production inventory model considering carbon cap from the perspective of supply chain integration with uncertain demand. Shen et al. [13] further targeted deteriorating items and established a production inventory model based on carbon tax policy. Under the trend of global carbon reduction initiatives, Mashud et al. [32], Paul et al. [33], Ruidas et al. [34], Pan et al. [35], and Huang et al. [16] tried to propose an EOQ/EPQ model or production inventory model with carbon emission reduction technology investment under various carbon emission reduction policies. Jauhari et al. [36] and Muthusamy et al. [37] further established a production inventory model to explore the impact of carbon taxes, green incentives, and green investments on improving supply chain carbon emissions.

Nevertheless, the above-mentioned production inventory models with carbon emission issues all considered that supply chain members face the same carbon emission reduction regulations. With the development trend of globalization, multi-national supply chains are commonly seen, which implies that the upstream and downstream members of the supply chain may come from different countries, and therefore may face different carbon emission reduction policies. In addition, Shen et al. [20] suggested that the government should adopt a differentiated hybrid carbon policy, especially for industries with high carbon emissions. Therefore, this study not only considers a multinational supply chain inventory management system with different carbon emission policy combinations, but also takes into account the hybrid carbon emission policy that high-carbon emission companies may face.

2.2. Inventory Model Using Game Theory

In order to explore the interaction between supply chain members in inventory-related decisions, Emmons and Gilbert [21] earlier used Stackelberg game theory to develop an inventory model considering a single supplier and single retailer and determined the supplier's wholesale price and return price as well as the retailer's order quantity. Following this, Chang et al. [22] discussed the supply chain inventory problem based on the Stackelberg game, where the seller, as a leader, determines the optimal replenishment period and the retailer, as a follower, determines the optimal ordering quantity. Chern et al. [23,24] and Jaggi et al. [25] applied the concepts of Stackelberg and Nash games, respectively, to establish supply chain inventory models with a permissible delay in payments. Wu et al. [12] further considered that demand is dependent on the period of delayed payment, and used the Stackelberg game to explore the interaction between delayed payment and ordering policy of the buyer and vendor. In addition, Lu et al. [26,27] considered carbon emission reduction investment options and used Stackelberg game theory to develop a sustainable production inventory model. Ma et al. [38] also took the carbon tax and government subsidies into consideration and used the Stackelberg game to make joint decisions among the manufacturer and retailer, and further developed the nested genetic algorithm to solve the model. Mahmoodi [39] and Mahdavisharif et al. [40] used Stackelberg game theory to establish a manufacturer–retailer supply chain inventory model for deteriorating items to determine the retail price and replenishment cycle. Xin al. [41] comparatively studied the purchasing strategies, pricing decisions and incentive mechanisms of different replenishment cycles, and derived the optimal decision on demand for green products based on the Stackelberg game. Recently, Choudhury al. [42] extended to a multilayer sustainable production inventory supply chain model with deteriorating items and pollution via Stackelberg game approach.

Although many studies have investigated various manufacturer–retailer supply chain inventory models, there is a research gap in the literature regarding the optimal equilibrium strategy for multinational supply chain members with both carbon emission policy combinations and mixed carbon emission policy. Based on the above, the main contributions of this study are as follows. First, the proposed model extends the production inventory problem to a multinational supply chain system carbon emission reduction investment under various carbon emission reduction policy combinations, thereby filling the research gap in previous literature. Second, this study uses game theory to explore the interaction of production, carbon reduction investments, and replenishment decisions between the manufacturer and retailer. Finally, this study uses numerical examples and sensitivity analysis to obtain some management insights and assist supply chain members in making decisions that can achieve both low-carbonization and maximum profit goals.

3. Notation and Assumptions

In order to set up the schema, we need the following notation, which is described as follows.

D	:	Demand rate
P	:	Production rate
A	:	Retailer's order cost/time
S	:	Manufacturer's setup cost/time
c	:	Manufacturer's production cost/unit
v	:	Manufacturer's supply price/unit
v	:	Retailer's selling price/unit
h_b	:	Retailer's holding cost/unit/unit time
h_v	:	Manufacturer's holding cost/unit/unit time
C_T	:	Ratailer's fixed delivery cost/time
C_t	:	Ratailer's variable delivery cost/unit
$\hat{A}$	:	Retailer's carbon emissions generated by ordering activities/time
$\hat{S}$	:	Manufacturer's carbon emissions generated by setup activities/time
$\hat{c}$	:	Manufacturer's carbon emissions generated by production activities/unit
$\hat{v}$	:	Retailer's carbon emissions generated by purchasing activities/unit
$\hat{h}_b$	:	Retailer's carbon emissions generated by holding activities/unit/unit time
$\hat{h}_v$	:	Manufacturer's carbon emissions generated by holding activities/unit/unit time
$\hat{C}_T$	:	Retailer's fixed carbon emissions generated by delievery activies/time
$\hat{C}_t$	:	Retailer's carbon emissions generated by delievery activies/unit
C_1	:	Carbon price in carbon trading market/unit
C_2	:	Carbon tax/unit
$\overline{\varpi}_b$	:	Retailer's total carbon emissions quota/unit time
$\overline{\varpi}_v$	:	Manufacturer's total carbon emissions quota/unit time
T_b	:	Retailer's length of a replenishment cycle, a decision variable
T_v	:	Manufacturer's length of a production cycle, a decision variable
T_s	:	Manufacturer's length of production period within a production cycle, is a decision variable
n	:	Number of times the manufacturer ships items to a retailer in a production cycle, an integer decision variable
q	:	Amount of items shipped by the manufacturer to the retailer at one time during a production cycle, a decision variabe
Q	:	Retailer's ordering quantity, a decision variabe
ξ	:	Investments to reduce carbon emissions, a decision variable
$m(\xi)$	:	Proportion of carbon emissions reduction, a function of ξ

Next, the main assumptions of the production inventory models discussed in this study are as follows:

1. A multinational supply chain system consisting of a single retailer and a single manufacturer from different countries is considered in which the retailer and the manufacturer face different carbon emission reduction policies. Further, the manufacturer may face hybrid carbon emission policies at the same time due to the characteristics of its high carbon emission industry.
2. This study explores a Stackelberg game in which the manufacturer is the leader and the retailer is the follower (see, for example, [23,26,38]).
3. The manufacturer's production rate is finite and greater than the demand rate; otherwise, the supply chain system will always be out of stock.
4. In terms of delivery strategy, assuming the same as Shen et al. [13], the retailer signs a contract with the manufacturer that requires the manufacturer to ship orders to the retailer in installments of a fixed quantity each time, and the cost of shipping is borne by the retailer.
5. The retailer's carbon emissions are related to the business activities of ordering, holding, transporting, and purchasing goods (please refer to [5,8,11]), while the manufacturer's carbon emissions are related to operational activities such as material procurement, setup, production, and storage (please refer to [9]).
6. Carbon emissions can be reduced through technological investments shared by the manufacturer and the retailer, with the reduced carbon emission rate being $m(\xi)$, where $0 < m(\xi) < 1$ and $m(\xi)$ is an increasing function of the investment in carbon emission technology ξ. Further, the proportions of capital investments by the retailer and manufacturer in technology for reducing carbon emissions are α and $1 - \alpha$, respectively, where $0 \leq \alpha \leq 1$.
7. When considering carbon tax policy, the retailer or the manufacturer is levied based on their respective carbon emissions. That is, carbon tax is levied in the form of a unit tax which is the same as Cheng et al. [18], Shen et al. [20], Lu et al. [26], and so on.
8. Similar to Wu et al. [12], Shen et al. [13], and Lu et al. [26,27], neither the retailer nor the manufacturer is allowed to be out of stock.

4. Model Formulation and Solution

From the perspective of the Stackelberg game, this study explores the following different carbon emission reduction policies faced by a single manufacturer and a single retailer. (1) The retailer faces a carbon tax policy, while the manufacturer faces a mixed policy of carbon cap-and-trade and carbon tax; (2) retailers face a carbon cap-and-trade policy, while manufacturers face a mixed policy of carbon cap-and-trade and carbon tax, as well as their respective production, inventory, and carbon reduction investment issues. According to the above symbols and assumptions, firstly, the total profit and carbon emission functions of the seller and the manufacturer are established, respectively, which are described as follows.

4.1. Retailer's Total Profit and Carbon Emissions

Based on the above notation and assumptions, the retailer's total profit is equal to sales revenue minus ordering cost, purchase cost, holding cost, shipping cost, and investment in carbon emissions. Since the length of a replenishment cycle is T_b, the retailer's total profit per unit of time (denoted by $TP_b(T_b,\ \xi)$) can be obtained as follows:

$$TP_b(T_b, \xi) = \left\{ \begin{array}{c} \text{Sales revenue} \\ -\text{Ordering cost} \\ -\text{Purchase cost} \\ -\text{Holding cost} \\ -\text{Shipping cost} \\ -\text{Investment in carbon emissions} \end{array} \right\} / T_b$$
$$= \left[pDT_b - A - vDT_b - \frac{h_b D T_b^2}{2} - C_T - C_t DT_b - \alpha\xi \right] / T_b \tag{1}$$

Moreover, according to assumption (6), the carbon emission of the retailer will be related to the operational activities such as ordering, purchase of goods, storage, and delivery, so the retailer's total carbon emissions per unit time (denoted by $E_b(T_b, \xi)$) can be obtained as follows:

$$E_b(T_b, \xi) = [1 - m(\xi)] \left\{ \begin{array}{c} \text{Carbon emissions generated by ordering activities} \\ +\text{Carbon emissions generated by purchase activities} \\ +\text{Carbon emissions generated by shipping activities} \\ +\text{Carbon emissions generated by holding activities} \end{array} \right\} / T_b$$
$$= [1 - m(\xi)] \left[\hat{A} + \hat{v} DT_b + \frac{\hat{h}_b D T_b^2}{2} + \hat{C}_T + \hat{C}_t DT_b \right] / T_b. \tag{2}$$

4.2. Manufacturer's Total Profit and Carbon Emissions

The manufacturer's total profit is equal to sales revenue minus setup costs, production costs, and holding costs; since the length of a production cycle is $T_v = T_p + (n-1)T_b$, the manufacturer's total profit per unit of time (denoted by $TP_v(T_v, T_s, \xi, n)$) can be obtained as follows:

$$TP_v(T_v, T_s, \xi, n) = \left\{ \begin{array}{c} \text{Sales revenue} - \text{Setup cost} - \text{Production cost} \\ -\text{Holding cost} - \text{Investment in carbon emissions} \end{array} \right\} / T_v$$
$$= \left\{ vnDT_b - S - cPT_s - \frac{h_v n D^2 T_b^2}{2} \left[\frac{2-n}{P} + \frac{(n-1)}{D} \right] - (1-\alpha)\xi \right\} / T_v. \tag{3}$$

Similarly, according to assumption (6), the carbon emissions of the manufacturer will be related to operational activities such as material procurement, installation, production, and storage, so the total carbon emissions per unit time of the manufacturer (denoted by $E_v(T_v, T_s, \xi, n)$) can be obtained as follows:

$$E_v(T_v, T_s, \xi, n) = [1 - m(\xi)] \left\{ \begin{array}{c} \text{Carbon emissions generated by setup activities} \\ +\text{Carbon emissions generated by production activities} \\ +\text{Carbon emissions generated by holding activities} \end{array} \right\} / T_v$$
$$= [1 - m(\xi)] \left\{ \hat{S} + \hat{c}PT_s + \frac{\hat{h}_v n D^2 T_b^2}{2} \left[\frac{2-n}{P} + \frac{(n-1)}{D} \right] \right\} / T_v. \tag{4}$$

Because of the length of time the manufacturer is engaged in production in a production cycle $T_s = nDT_b/P$, and the length of the production cycle $T_v = DT_b/P + (n-1)T_b$, $TP_v(T_v, T_s, \xi, n)$ and $E_v(T_v, T_s, \xi, n)$ can be reduced, respectively, to $TP_v(T_b, \xi, n)$ and $E_v(T_b, \xi, n)$.

Next, considering the following situations: (I) retailers facing a carbon tax policy and manufacturers facing a mixed policy of carbon cap-and-trade and carbon tax, (II) where the retailer faces a cap-and-trade policy and the manufacturer faces a mixed policy of cap-and-trade and carbon tax. The purpose of this study is to determine the manufacturer's production, delivery, and investing strategies and the retailer's replenishment strategy, respectively, so as to maximize the individual total profit at equilibrium. If the enterprise retailer or manufacturer belongs to a country that has established a carbon cap-and-trade mechanism, its total carbon emission quota will be ω_b or ω_b. When its carbon emissions exceed or fall below the quota, it will be traded at price C_1 through the carbon trading market to sell or obtain additional rights. Relatively speaking, if the country where the supply chain members belong adopts a carbon tax policy, their government agencies will induce them to consider environmental costs through additional carbon taxes. To simplify the model, this study considers that the tax schedule is linear, that is, companies pay C_2 per unit of carbon emissions [5].

For the above two situations, this study will calculate the total profit per unit time of the retailer and the manufacturer, respectively, under the carbon emission policies (denoted by $\Pi_b^{\mathrm{I}}(T_b,\ \xi)$, $\Pi_b^{\mathrm{II}}(T_b,\ \xi)$, and $\Pi_v(T_b,\ \xi,\ n)$, respectively) as follows:

$$\begin{aligned}\Pi_b^{\mathrm{I}}(T_b,\ \xi) &= TP_b(T_b,\ \xi) - C_1[E_b(T_b,\ \xi) - \overline{\omega}_b] \\ &= \tfrac{1}{T_b}\left[pDT_b - \{A + C_1[1-m(\xi)]\hat{A}\} - \{v + C_1[1-m(\xi)]\hat{v}\}T_b - \right. \\ &\quad \tfrac{\left\{h_b + C_1[1-m(\xi)]\hat{h}_b\right\}DT_b^2}{2} - \{C_T + C_1[1-m(\xi)]\hat{C}_T\} - \{C_t + \\ &\quad \left. C_1[1-m(\xi)]\hat{C}_t\right\}DT_b] + C_1\overline{\omega}_b,\end{aligned} \tag{5}$$

$$\begin{aligned}\Pi_b^{\mathrm{II}}(T_b,\ \xi) &= TP_b(T_b,\ \xi) - C_2 E_b(T_b,\ \xi) \\ &= \tfrac{1}{T_b}\left[pDT_b - \{A + C_2[1-m(\xi)]\hat{A}\} - \{v + C_2[1-m(\xi)]\hat{v}\}T_b - \right. \\ &\quad \tfrac{\left\{h_b + C_2[1-m(\xi)]\hat{h}_b\right\}DT_b^2}{2} - \{C_T + C_2[1-m(\xi)]\hat{C}_T\} - \{C_t + \\ &\quad \left. C_2[1-m(\xi)]\hat{C}_t\right\}DT_b]\end{aligned} \tag{6}$$

$$\begin{aligned}\Pi_v(T_b,\ \xi,\ n) &= TP_v(T_b,\ \xi,\ n) - C_1[E_v(T_b,\ \xi,\ n) - \overline{\omega}_v] - C_2 E_v(T_b,\ \xi,\ n) \\ &= \tfrac{1}{(DT_b/P)+(n-1)T_b}\left\{vnDT_b - \{S + (C_1 + C_2)[1-m(\xi)]\hat{S}\} - \{c + (C_1\right. \\ &\qquad +C_2)\,[1-m(\xi)]\hat{c}\}nDT_b \\ &\qquad \left. - \tfrac{\{h_v + (C_1+C_2)\,[1-m(\xi)]\hat{h}_v\}nD^2T_b^2}{2}\left[\tfrac{2-n}{P} + \tfrac{(n-1)}{D}\right]\right\} \\ &\qquad + C_1\overline{\omega}_v\end{aligned} \tag{7}$$

In view of the above different scenarios, this study intends to solve the problem from the perspective of a Stackelberg game, with the main purpose of seeking the manufacturer's optimal production, delivery, investing policies, and the retailer's optimal ordering policy to achieve game equilibrium under different carbon emission policy combinations, so as to maximize their respective total profit functions. The description of the Stackelberg game is as follows.

4.3. Stackelberg Game and Equilibrium

Consider a Stackelberg game with two players involved, the manufacturer and the retailer, in which the manufacturer is a leader, and the retailer is a follower. Suppose the manufacturer's strategy is $(x,\ y)$ and the retailer's strategy is z. The two sides face the profit functions of $f_A(x,\ y,\ z)$ and $f_B(x,\ y,\ z)$, respectively, where $(x,\ y,\ z) \in A \times B$. A and B is defined as the set of strategies that the manufacturer and the retailer can take and $A \times B$ is defined as the set of all reasonable decisions. The purpose is to find a solution $(x,\ y, z) \in A \times B$ that maximizes the manufacturer's and the retailer's profit functions ($f_A(x,\ y, z)$ and $f_B(x,\ y, z)$). Based on the fact that the manufacturer is the leader, the retailer is the follower, and the manufacturer's and the retailer's strategies are $(x,\ y)$ and z, the manufacturer's goal is to maximize $f_A(x,\ y,\ z)$ and the retailer's goal is to maximize $f_B(x,\ y,\ z)$ for any given $(x,\ y)$. Accordingly, $(x^*,\ y^*,\ z^*)$ represents Stackelberg equilibrium if it satisfies the following conditions (Lu et al. [26,27]):

$$\begin{cases} f_A(x^*, y^*,\ z^*) = sup f_A(x, y, Z(x, y)), & x \in A, \\ Z(x, y) = max f_B(x, y, z), & y \in B, \end{cases}$$

The process of solving the Stackelberg equilibrium is explained as follows. First, the retailer (i.e., the follower) finds its value of T_b to maximize its own profit, which is a function of ξ $(T_b(\xi))$. Then the manufacturer (i.e., the leader) determined the optimal values of $(n,\ \xi)$ based on the value of $T_b(\xi)$. Once the optimal values of $(n,\ \xi)$ are determined, the optimal value of T_b is obtained, thereby achieving the Stackelberg equilibrium. Due to the complexity of the model and the fact that n is an integer, it is difficult to find the close form of $(n,\ \xi)$ and check the concavity of the manufacturer's profit function directly. Alternatively, this study develops a simple algorithm (Algorithm 1) to obtain the solutions for the manufacturer and the retailer under a Stackelberg equilibrium.

Algorithm 1. The process of finding the optimal equilibrium solution.

Step 1. Solve $\frac{\partial \Pi_b^{\mathrm{I}}(T_b,\ \xi)}{\partial T_b} = 0$ or $\frac{\partial \Pi_b^{\mathrm{II}}(T_b,\ \xi)}{\partial T_b} = 0$ to find the optimal value of T_b (say $T_b^{\mathrm{I}}(\xi)$ or $T_b^{\mathrm{II}}(\xi)$ which is function of ξ) and then substitute $T_b(\xi)$ into (7) to obtain $\Pi_v\left(T_b,\ \xi,\ n|T_b = T_b^{\mathrm{I}}(\xi)\right)$ or $\Pi_v\left(T_b,\ \xi,\ n|T_b = T_b^{\mathrm{II}}(\xi)\right)$.

Step 2. Let $n = 1$.

Step 3. Find the value of ξ (say $\xi_{(n)}^{\mathrm{I}}$ or $\xi_{(n)}^{\mathrm{II}}$) by setting $\frac{\partial \Pi_v\left(T_b, \xi, n|T_b = T_b^{\mathrm{I}}(\xi)\right)}{\partial \xi} = 0$ or $\frac{\partial \Pi_v\left(T_b, \xi, n|T_b = T_b^{\mathrm{II}}(\xi)\right)}{\partial \xi} = 0$.

Step 4. Substitute $\xi_{(n)}^{\mathrm{I}}$ or $\xi_{(n)}^{\mathrm{II}}$ into (7) to obtain $\Pi_v\left(T_b, \xi_{(n)}^{\mathrm{I}}, n\middle|T_b = T_b^{\mathrm{I}}\left(\xi_{(n)}^{\mathrm{I}}\right)\right)$ or $\Pi_v\left(T_b, \xi_{(n)}^{\mathrm{II}}, n\middle|T_b = T_b^{\mathrm{II}}\left(\xi_{(n)}^{\mathrm{II}}\right)\right)$.

Step 5. Set $n = n + 1$, and repeat Step 3 to get $\Pi_v\left(T_b, \xi_{(n+1)}^{\mathrm{I}}, n+1\middle|T_b = T_b^{\mathrm{I}}\left(\xi_{(n+1)}^{\mathrm{I}}\right)\right)$ or $\Pi_v\left(T_b, \xi_{(n+1)}^{\mathrm{II}}, n+1\middle|T_b = T_b^{\mathrm{II}}\left(\xi_{(n+1)}^{\mathrm{II}}\right)\right)$.

Step 6. If $\Pi_v\left(T_b, \xi_{(n+1)}^{\mathrm{I}}, n+1\middle|T_b = T_b^{\mathrm{I}}\left(\xi_{(n+1)}^{\mathrm{I}}\right)\right) < \Pi_v\left(T_b, \xi_{(n)}^{\mathrm{I}}, n\middle|T_b = T_b^{\mathrm{I}}\left(\xi_{(n)}^{\mathrm{I}}\right)\right)$ or $\Pi_v\left(T_b, \xi_{(n+1)}^{\mathrm{II}}, n+1\middle|T_b = T_b^{\mathrm{II}}\left(\xi_{(n+1)}^{\mathrm{II}}\right)\right) < \Pi_v\left(T_b, \xi_{(n)}^{\mathrm{II}}, n\middle|T_b = T_b^{\mathrm{II}}\left(\xi_{(n)}^{\mathrm{II}}\right)\right)$, then $\Pi_v\left(T_b, \xi^*, n^*|T_b = T_b(\xi^*)\right) = \Pi_v\left(T_b, \xi_{(n)}^{\mathrm{I}}, n\middle|T_b = T_b^{\mathrm{I}}\left(\xi_{(n)}^{\mathrm{I}}\right)\right)$ or $\Pi_v\left(T_b, \xi_{(n)}^{\mathrm{II}}, n\middle|T_b = T_b^{\mathrm{II}}\left(\xi_{(n)}^{\mathrm{II}}\right)\right)$ and hence $(\xi^*, n^*) = \left(\xi_{(n)}^{\mathrm{I}}, n\right)$ or $\left(\xi_{(n)}^{\mathrm{II}}, n\right)$ is the optimal solution for the vendor. Otherwise, return to Step 5.

Step 7. Substitute (n^*, ξ^*) into $\frac{\partial \Pi_b^{\mathrm{I}}(T_b, \xi)}{\partial T_b} = 0$ or $\frac{\partial \Pi_b^{\mathrm{II}}(T_b, \xi)}{\partial T_b} = 0$ and solve it to find the optimal value of $T_b^* = T_b(\xi^*)$.

Once the equilibrium solution $\left(T_b^*, \xi^*, n^*\right)$ is obtained, we can determine the total profits and the amount of carbon emissions produced by the retailer and the manufacturer $E_b\left(T_b^*, \xi^*\right)$ and $E_v\left(T_b^*, \xi^*, n^*\right)$. Further, the corresponding maximum profits can be calculated as $\Pi_b^{\mathrm{I}}\left(T_b^*, \xi^*\right)$ or $\Pi_b^{\mathrm{II}}\left(T_b^*, \xi^*\right)$ and $\Pi_v\left(T_b^*, \xi^*, n^*\right)$.

5. Numerical Analysis

5.1. Numerical Examples

This section considers a multi-national supply chain system and uses several numerical examples, drawing partly from previous literature, such as Ghosh et al. [43] and Lu et al. [26,27], and partly utilizing reasonable values to verify the proposed production inventory models. The relevant parameter values are described in Table 1 as follows.

Table 1. The values of all parameters used in the proposed model.

Parameter	Value
Demand rate	2000 units/year
Production rate	6000 units/year
Retailer's ordering cost	USD 200/order
Manufacturer's setup cost	USD 500/setup
Manufacturer's production cost	USD 15/unit
Manufacturer's supply price	USD 25/unit
Retailer's selling price	USD 80/unit
Retailer's holding cost	USD 0.5/unit/year
Manufacturer's holding cost	USD 0.3/unit/year
Ratailer's fixed delivery cost	USD 50/shipment
Ratailer's variable delivery cost	USD 3/unit
Retailer's carbon emissions generated by ordering activities	50 kg/order
Manufacturer's carbon emissions generated by setup activities	150 kg/setup
Manufacturer's carbon emissions generated by production activities	0.8 kg/unit
Retailer's carbon emissions generated by purchasing activities	1 kg/unit
Retailer's carbon emissions generated by holding activities	0.05 kg/unit/year
Manufacturer's carbon emissions generated by holding activities	0.03 kg/unit/year
Retailer's fixed carbon emissions generated by delievery activies	3 kg/time
Retailer's carbon emissions generated by delievery activies/unit	0.05 kg/unit
Carbon price in carbon trading market/unit	USD 10/unit
Carbon tax	USD 8/unit
Retailer's total carbon emissions quota	1500 kg/year
Manufacturer's total carbon emissions quota	1500 kg/year
Proportion of carbon emissions reduction	$\frac{1}{3}\left(1-e^{-0.01\xi}\right)$

Based on the above-mentioned calculation method and the mathematical software Mathematica 7.0, we illustrate the solution processes and results for the two different situations.

Situation I: The retailer faces a carbon tax policy, and the manufacturer faces a mixed policy of carbon cap-and-trade and carbon tax.

In this situation, when the Stackelberg equilibrium is reached, the optimal number of times for the manufacturer to ship the item to the retailer n^* = 4 and the optimal carbon reduction investment amount ξ^* = $568.715. The optimal replenishment cycle length of the retailer T_b^* = 0.6295 years and optimal order quantity Q^* = 5035.64 units. At this time, the optimal carbon emission quantity and total profit of the manufacturer and the retailer to achieve Stackelberg equilibrium are E_b^* = 1500.64 kg, E_v^* = 1050.17 kg, Π_b^{I*} = $102,877, and Π_v^{I*} = $13,781. The solving process for Situation I is illustrated in Table 2 below.

Table 2. Solution process of the proposed model (Situation I).

n	T_b	ξ	q	Q	E_b	E_v	Π_b^{I}	Π_v
1	0.6243	458.295	1248.65	1248.65	1505.88	927.97	102,843	12,154
2	0.6271	518.471	1254.21	2508.42	1502.35	996.16	102,868	13,390
3	0.6286	550.302	1257.18	3771.55	1501.17	1028.67	102,874	13,726
4	0.6295	568.715	1258.91	5035.64	1500.64	1050.17	102,877	13,781←
5	0.6299	579.242	1259.90	6299.49	1500.37	1066.82	102,878	13,712
6	0.6302	584.761	1260.42	7562.50	1500.24	1080.92	102,878	13,577

Note: The symbol "←" represents the optimal equilibrium solution.

Scenario II: The retailer faces a cap-and-trade policy, and the manufacturer faces a mixed policy of cap-and-trade and a carbon tax.

In this situation, when the Stackelberg equilibrium is reached, the optimal number of times for the manufacturer to ship the item to the retailer n^* = 4 and the optimal carbon reduction investment amount ξ^* = $568.315. The optimal replenishment cycle length of the retailer $T_b^* = 0.6201$ years and optimal order quantity $Q^* = 4960.91$ units. At this time, the optimal carbon emission quantity and total profit of the manufacturer and the retailer to achieve Stackelberg equilibrium are $E_b^* = 1500.87$ kg, $E_v^* = 1050.33$ kg, $\Pi_b^{II^*} = \$90{,}877.1$, and $\Pi_v^{I^*} = \$13{,}779$. Similarly, the solving process for Situation II is illustrated in Table 3 below.

Table 3. Solution process of the proposed model (Scenario II).

n	T_b	ξ	q	Q	E_b	E_v	Π_b^{II}	Π_v
1	0.6146	460.600	1229.14	1229.14	1505.95	929.73	90,856.1	12,103
2	0.6176	520.131	1235.24	2470.48	1502.51	997.00	90,873.3	13,366
3	0.6192	551.018	1238.44	3715.31	1501.37	1029.10	90,877.1	13,715
4	0.6201	568.315	1240.23	4960.91	1500.87	1050.33	90,878.1	13,779←
5	0.6206	577.689	1241.20	6206.00	1500.63	1066.77	90,878.4	13,716
6	0.6208	582.114	1241.66	7449.96	1500.52	1080.69	90,878.5	13,588

Note: The symbol "←" represents the optimal equilibrium solution.

Next, we will compare the carbon emissions and total profits of the two situations under the Stackelberg equilibrium. The following tables show comparisons of the optimal carbon emissions and total profits of Situation I and Situation II considering different carbon price, C_1, and carbon tax, C_2 (see Tables 4 and 5).

Table 4. Comparison of optimal carbon emissions and total profit under various carbon prices.

C_1	**Situation I**				**Situation II**			
	E_b	E_v	Π_b^{I}	Π_v	E_b	E_v	Π_b^{II}	Π_v
8	1501.22	1050.57	102,877.4	12,880	1501.22	1050.57	90,877.4	12,880
9	1500.91	1050.36	102,876.9	13,330	1501.04	1050.44	90,877.8	13,329
10	1500.64	1050.17	102,876.7	13,781	1500.87	1050.33	90,878.1	13,779
11	1500.39	1050.00	102,876.8	14,232	1500.72	1050.22	90,878.3	14,228
12	1500.18	1049.86	102,877.1	14,683	1500.58	1050.13	90,878.5	14,678

Table 5. Comparison of optimal carbon emissions and total profits under various carbon taxes.

C_2	**Situation I**				**Situation II**			
	E_b	E_v	Π_b^{I}	Π_v	E_b	E_v	Π_b^{II}	Π_v
6.4	1500.91	1050.36	102,875.6	15,461	1501.39	1050.69	93,280	15,457
7.2	1500.77	1050.26	102,876.2	14,621	1501.12	1050.50	92,079	14,618
8	1500.64	1050.17	102,876.7	13,781	1500.87	1050.33	90,878	13,779
8.8	1500.52	1050.09	102,877.1	12,941	1500.65	1050.17	89,678	12,939
9.6	1500.41	1050.01	102,877.4	12,101	1500.45	1050.04	88,478	12,100

From the results of Tables 4 and 5, it can be found that the carbon emissions of retailers and manufacturers will decrease regardless of the increase in carbon price or carbon tax. In terms of total profit, when the carbon price increases, the total profit of manufacturers will increase; conversely, the total profit of manufacturers will decrease as the carbon tax increases. As to the retailer, if the carbon tax is increased, its total profit will be reduced, aligning with previous related studies. However, the difference is that the retailer's optimal total profit will first decrease and then increase in Situation I, while in Situation II, the retailer's optimal total profit will decrease as the carbon price increases. Further, through comparison, it is found that the carbon emissions of the manufacturer and the retailer in Situation II are higher than those in Situation I, while the total profits of the manufacturer and the retailer in Situation I are higher than those in Situation II.

5.2. Sensitivity Analysis

To comprehend the impact of the changes in the parameters of the production inventory model of the supply chain system on the decision making, carbon emissions, and total profit among its members, this study conducts a sensitivity analysis of the model's parameters. It integrates and elaborates on the numerical analysis results to furnish a reference for the manufacturer and the retailer, aiding them in decision making to adapt to external environmental changes. Given the extensive number of model parameters, this study will be divided into two groups: those related to total profit and those related to carbon emissions. The sensitivity analysis results of each parameter are displayed in Tables 6 and 7.

Table 6. Optimal solution under changes in parameters related to total profit.

Parameter	Value	n^*	T_b^*	ξ^*	q^*	Q^*	E_b^*	E_v^*	Π_b^{I*}	Π_v^*
	1600	4	0.7031	557.115	1125.03	4500.12	1210.05	860.98	85,102	13,778
	1800	4	0.6632	563.193	1193.82	4775.26	1355.49	956.45	93,986	13,775
D	2000	4	0.6295	568.715	1258.91	5035.64	1500.64	1050.17	102,877	13,781
	2200	4	0.6004	573.784	1320.85	5283.42	1645.53	1142.21	111,772	13,794
	2400	4	0.5750	578.482	1380.07	5520.28	1790.20	1232.65	120,673	13,813
	4800	4	0.62952	570.048	1259.04	5036.14	1500.60	1028.43	102,877	13,867
	5400	4	0.62948	569.304	1258.97	5035.86	1500.62	1040.40	102,877	13,819
P	6000	4	0.62946	568.715	1258.91	5035.64	1500.64	1050.17	102,877	13,781
	6600	4	0.62943	568.236	1258.87	5035.46	1500.65	1058.28	102,877	13,749
	7200	4	0.62941	567.839	1258.83	5035.31	1500.66	1065.13	102,877	13,722
	160	4	0.6100	567.152	1220.06	4880.25	1501.17	1050.53	102,941	13,776
	180	4	0.6198	567.931	1239.64	4958.55	1500.89	1050.34	102,909	13,779
A	200	4	0.6295	568.715	1258.91	5035.64	1500.64	1050.17	102,877	13,781
	220	4	0.6389	569.501	1277.89	5111.57	1500.41	1050.02	102,845	13,783
	240	4	0.6483	570.288	1296.60	5186.39	1500.22	1049.89	102,814	13,784
	400	4	0.62942	567.854	1258.83	5035.32	1500.66	1050.18	102,877	13,818
	450	4	0.62944	568.283	1258.87	5035.48	1500.65	1050.18	102,877	13,799
S	500	4	0.62946	568.715	1258.91	5035.64	1500.64	1050.17	102,877	13,781
	550	4	0.62948	569.148	1258.95	5035.80	1500.62	1050.16	102,877	13,763
	600	4	0.62950	569.583	1258.99	5035.97	1500.61	1050.15	102,877	13,744
	12	5	0.62995	579.242	1259.90	6299.49	1500.37	1066.82	102,878	19,337
	13.5	4	0.62946	568.715	1258.91	5035.64	1500.64	1050.17	102,877	16,550
c	15	4	0.62946	568.715	1258.91	5035.64	1500.64	1050.17	102,877	13,781
	16.5	3	0.62859	550.302	1257.18	3771.55	1501.17	1028.67	102,874	11,026
	18	3	0.62859	550.302	1257.18	3771.55	1501.17	1028.67	102,874	8326
	20	3	0.62859	550.302	1257.18	3771.55	1501.17	1028.67	112,874	4726
	22.5	3	0.62859	550.302	1257.18	3771.55	1501.17	1028.67	107,874	9226
v	25	4	0.62946	568.715	1258.91	5035.64	1500.64	1050.17	102,877	13,781
	27.5	4	0.62946	568.715	1258.91	5035.64	1500.64	1050.17	102,877	13,781
	30	5	0.62995	579.242	1259.90	6299.49	1500.37	1066.82	92,878	23,087
	0.4	4	0.6712	572.295	1342.30	5369.20	1499.84	1049.62	103,007	13,786
	0.45	4	0.6493	570.485	1298.61	5194.44	1500.20	1049.87	102,941	13,784
h_b	0.5	4	0.6295	568.715	1258.91	5035.64	1500.64	1050.17	102,877	13,781
	0.55	4	0.6113	566.987	1222.63	4890.51	1501.14	1050.51	102,814	13,776
	0.6	4	0.5947	565.303	1189.30	4757.20	1501.69	1050.88	102,754	13,770
	0.24	4	0.62955	570.652	1259.09	5036.37	1500.58	1050.13	102,877	13,862
	0.27	4	0.62950	569.679	1259.00	5036.00	1500.61	1050.15	102,877	13,822
h_v	0.3	4	0.62946	568.715	1258.91	5035.64	1500.64	1050.17	102,877	13,781
	0.33	4	0.62941	567.760	1258.82	5035.28	1500.66	1050.19	102,877	13,740
	0.36	4	0.62937	566.815	1258.73	5034.93	1500.69	1050.20	102,876	13,700

Table 6. *Cont.*

Parameter	Value	n^*	T_b^*	ξ^*	q^*	Q^*	E_b^*	E_v^*	Π_b^{I*}	Π_v^*
C_t	2.4	4	0.6295	568.715	1258.91	5035.64	1500.64	1050.17	104,077	13,781
	2.7	4	0.6295	568.715	1258.91	5035.64	1500.64	1050.17	103,477	13,781
	3	4	0.6295	568.715	1258.91	5035.64	1500.64	1050.17	102,877	13,781
	3.3	4	0.6295	568.715	1258.91	5035.64	1500.64	1050.17	102,277	13,781
	3.6	4	0.6295	568.715	1258.91	5035.64	1500.64	1050.17	101,677	13,781
C_T	40	4	0.6247	568.322	1249.31	4997.24	1500.76	1050.25	102,893	13,779.7
	45	4	0.6271	568.518	1254.12	5016.48	1500.70	1050.21	102,885	13,780.3
	50	4	0.6295	568.715	1258.91	5035.64	1500.64	1050.17	102,877	13,780.8
	55	4	0.6318	568.911	1263.68	5054.73	1500.58	1050.13	102,869	13,781.3
	60	4	0.6342	569.108	1268.44	5073.75	1500.52	1050.09	102,861	13,781.8
p	64	4	0.6295	568.715	1258.91	5035.64	1500.64	1050.17	70,877	13,781
	72	4	0.6295	568.715	1258.91	5035.64	1500.64	1050.17	86,877	13,781
	80	4	0.6295	568.715	1258.91	5035.64	1500.64	1050.17	102,877	13,781
	88	4	0.6295	568.715	1258.91	5035.64	1500.64	1050.17	118,877	13,781
	96	4	0.6295	568.715	1258.91	5035.64	1500.64	1050.17	134,877	13,781

Table 7. Optimal solution under changes in parameters related to carbon emissions.

Parameter	Value	n^*	T_b^*	ξ^*	q^*	Q^*	E_b^*	E_v^*	Π_b^{I*}	Π_v^*
$\hat{A}$	40	4	0.5967	566.195	1193.35	4773.41	1490.41	1050.82	102,985	13,771
	45	4	0.6133	567.436	1226.57	4906.27	1495.63	1050.46	102,930	13,777
	50	4	0.6295	568.715	1258.91	5035.64	1500.64	1050.17	102,877	13,781
	55	4	0.6452	570.017	1290.44	5161.77	1505.46	1049.93	102,824	13,784
	60	4	0.6606	571.330	1321.22	5284.89	1510.11	1049.74	102,773	13,785
$\hat{S}$	120	4	0.62928	564.934	1258.56	5034.22	1500.74	1042.89	102,876.3	13,913
	135	4	0.62937	566.813	1258.73	5034.93	1500.69	1046.53	102,876.5	13,847
	150	4	0.62946	568.715	1258.91	5035.64	1500.64	1050.17	102,876.7	13,781
	165	4	0.62955	570.639	1259.09	5036.36	1500.59	1053.81	102,876.8	13,715
	180	4	0.62964	572.588	1259.27	5037.09	1500.54	1057.44	102,877.0	13,649
$\hat{c}$	0.64	4	0.6285	547.657	1256.94	5027.75	1501.25	853.27	102,874.1	17,332
	0.72	4	0.6290	558.733	1257.97	5031.90	1500.91	951.72	102,875.6	15,556
	0.8	4	0.6295	568.715	1258.91	5035.64	1500.64	1050.17	102,876.7	13,781
	0.88	4	0.6299	577.798	1259.76	5039.05	1500.41	1148.62	102,877.4	12,006
	0.96	3	0.6294	567.696	1258.81	3776.44	1500.66	1220.63	102,876.6	10,264
$\hat{v}$	0.8	4	0.6295	568.715	1258.91	5035.64	1233.52	1050.17	105,548	13,781
	0.9	4	0.6295	568.715	1258.91	5035.64	1367.08	1050.17	104,212	13,781
	1	4	0.6295	568.715	1258.91	5035.64	1500.64	1050.17	102,877	13,781
	1.1	4	0.6295	568.715	1258.91	5035.64	1634.19	1050.17	101,541	13,781
	1.2	4	0.6295	568.715	1258.91	5035.64	1767.75	1050.17	100,205	13,781
$\hat{h}_b$	0.04	4	0.6564	571.075	1312.79	5251.17	1491.30	1049.78	102,963	13,785
	0.045	4	0.6425	569.881	1285.01	5140.03	1496.05	1049.97	102,919	13,783
	0.05	4	0.6295	568.715	1258.91	5035.64	1500.64	1050.17	102,877	13,781
	0.055	4	0.6172	567.574	1234.33	4937.33	1505.09	1050.39	102,835	13,778
	0.06	4	0.6056	566.459	1211.14	4844.55	1509.40	1050.63	102,794	13,774
$\hat{h}_v$	0.024	4	0.62954	570.523	1259.08	5036.32	1500.59	1044.7	102,876.84	13,879
	0.07	4	0.62950	569.613	1258.99	5035.98	1500.61	1047.44	102,876.75	13,830
	0.03	4	0.62946	568.715	1258.91	5035.64	1500.64	1050.17	102,876.67	13,781
	0.033	4	0.62941	567.830	1258.83	5035.31	1500.66	1052.90	102,876.58	13,732
	0.036	4	0.62937	566.957	1258.75	5034.98	1500.68	1055.63	102,876.50	13,683

Table 7. *Cont.*

Parameter	Value	n^*	T_b^*	ξ^*	q^*	Q^*	E_b^*	E_v^*	Π_b^{I*}	Π_v^*
$\hat{C}_t$	0.05	4	0.6295	568.715	1258.91	5035.64	1487.28	1050.17	103,010	13,781
	0.05	4	0.6295	568.715	1258.91	5035.64	1493.96	1050.17	102,943	13,781
	0.05	4	0.6295	568.715	1258.91	5035.64	1500.64	1050.17	102,877	13,781
	0.05	4	0.6295	568.715	1258.91	5035.64	1507.31	1050.17	102,810	13,781
	0.05	4	0.6295	568.715	1258.91	5035.64	1513.99	1050.17	102,743	13,781
$\hat{C}_T$	2.4	4	0.6275	568.560	1255.07	5020.29	1500.04	1050.20	102,883	13,780.4
	2.7	4	0.6285	568.637	1256.99	5027.97	1500.34	1050.18	102,880	13,780.6
	3	4	0.6295	568.715	1258.91	5035.64	1500.64	1050.17	102,877	13,780.8
	3.3	4	0.6304	568.792	1260.82	5043.30	1500.93	1050.15	102,873	13,781.0
	3.6	4	0.6314	568.870	1262.74	5050.94	1501.22	1050.14	102,870	13,781.2

Based on Table 6, the following numerical analysis results can be obtained:

(1) When the market demand rate increases, the retailer's optimal order quantity and the manufacturer's optimal carbon emission reduction investment will increase, and the retailer's optimal replenishment cycle will decrease. This increase will also lead to an increase in the carbon emissions and total profit of the retailer, as well as the carbon emissions of the manufacturer. Nevertheless, the total profit of the manufacturer will initially decrease and then increase as the demand rate rises.

(2) When the manufacturer's production rate increases, the retailer's optimal replenishment cycle and order quantity will decrease, and the manufacturer's optimal carbon reduction investment will also decrease. At this time, both the retailer's carbon emissions and the manufacturer's carbon emissions will increase. The manufacturer's total profit will decrease as its production rate increases, while the retailer's total profit will remain unaffected by this change.

(3) When the retailer's ordering cost, the retailer's fixed delivery cost, or the manufacturer's setup cost increases, the retailer's optimal replenishment cycle and order quantity will increase, and the manufacturer's optimal carbon emission reduction investment will also increase. This will result in both retailers' carbon emissions and manufacturers' carbon emissions being reduced. As generally recognized, an increase in the retailer's ordering cost or fixed delivery cost will reduce its total profit, while changes in the manufacturer's setup cost will not affect the retailer's total profit. Interestingly, the manufacturer's total profit increases as the retailer's ordering cost or fixed delivery cost increases.

(4) With an increase in the manufacturer's production cost, the retailer's optimal replenishment cycle and order quantity will decrease, and the manufacturer's optimal carbon emission reduction investment will also decrease. Consequently, the retailer's carbon emissions will increase and the manufacturer's carbon emissions will decrease; the manufacturer's total profit will decrease as production costs increase. Moreover, generally speaking, the retailer's total profit will not be affected by changes in the manufacturer's production costs if the manufacturer does not adjust its shipping strategy. However, when the production cost increases to a certain level (for example, $c = 18$ in Table 6), the retailer's profits will then decrease.

(5) When the manufacturer's supply price increases, the retailer's optimal replenishment cycle and order quantity will increase, and the manufacturer's optimal carbon emission reduction investment will also increase. This causes the retailer's carbon emissions to decrease, and the manufacturer's carbon emissions to increase. Further, with the increase in the manufacturer's supply price, the retailer's total profit will decrease, while the manufacturer's total profit will increase.

(6) Regarding the impact of changes in holding costs, whether the holding cost of the retailer or the manufacturer increases, the retailer's optimal replenishment cycle and order quantity will decrease, and the manufacturer's optimal carbon emission reduc-

tion investment will also decrease. Likewise, this results in increased carbon emissions for both the retailer and the manufacturer. However, the change in the retailer's or manufacturer's holding cost has different impacts on their total profits. That is, the total profits of the retailer and manufacturer will decrease as the retailer's holding cost increases. However, although the manufacturer's total profit will decrease as the manufacturer's holding cost increases, once the manufacturer's holding cost increases to a certain level (for example, h_v = 0.36 in Table 6), the retailer's profit will decrease accordingly.

(7) When the retailer's variable delivery cost increases or selling price decreases, the retailer's total profit decreases, yet it does not impact the retailer's optimal replenishment cycle and order quantity, the manufacturer's optimal carbon emission reduction investment, the retailer's and the manufacturer's carbon emission, and the manufacturer's total profit within the supply chain system, where the manufacturer acts as the leader.

According to Table 7, the following numerical analysis results can be obtained:

(1) As the carbon emissions generated by retailers' ordering activities or the fixed carbon emissions generated by retailers' delivery activities increase, the retailer's optimal replenishment cycle and order quantity will increase, and the manufacturer's optimal carbon emission reduction investment will also increase. This will lead to an increase in the retailer's carbon emissions but a decrease in those of the manufacturer. Consequently, the retailer's total profit will diminish, whereas the manufacturer's total profit will increase.

(2) When the carbon emissions generated by the manufacturer's setup activities increase, the retailer's optimal replenishment cycle and order quantity will increase, and the manufacturer's optimal carbon emission reduction investment will also increase. This will cause the manufacturer's carbon emissions to increase, but the retailer's carbon emissions will decrease and the total profit of the manufacturer will decrease, while the retailer's profit will increase accordingly.

(3) As the carbon emissions generated by the manufacturer's production activities increase, the retailer's optimal replenishment cycle and order quantity will increase, and the manufacturer's optimal carbon emission reduction investment will also increase. This causes the manufacturer's carbon emissions to increase, but the retailer's carbon emissions will increase first and then decrease. The manufacturer's total profit decreases as the carbon emissions generated by the manufacturer's production activities increases, while the retailer's profit increases. Furthermore, changes in carbon emissions caused by production activities (for example, $\hat{c} = 0.96$) may affect the manufacturer's shipping strategy.

(4) As the carbon emissions generated by retailers' purchase activities or the variable carbon emissions generated by retailers' delivery activities increase, only the retailer's carbon emissions will increase, but its total profit will decrease, which will not affect the retailer's optimal replenishment cycle and order quantity, nor the manufacturer's optimal carbon emission reduction investment, total carbon emissions, and total profit.

(5) When the unit carbon emission of the retailer's or manufacturer's holding activities increases, the retailer's optimal replenishment cycle and order quantity will decrease, and the manufacturer's optimal carbon emission reduction investment will also decrease. This results in higher carbon emissions for the retailer and the manufacturer but decreases their total profits.

6. Conclusions

This study mainly focused on production inventory problems of the manufacturers with high carbon emissions, addressing a mixed carbon emission policy of carbon cap-and-trade and carbon tax simultaneously, and explored different situations where the retailer may face carbon emission policies. Additionally, by considering a Stackelberg game in which the manufacturer is the leader and the retailer is the follower, this study determined the optimal production, shipping, carbon reduction investment, and replenishment for both parties under the equilibrium for different carbon emission policy combinations. In order to verify the model and explain the solution process, this study used several numerical examples to solve and compare the proposed models. Through comparison with numerical examples, it can be concluded that regardless of the increase in carbon price or carbon tax, the retailer and the manufacturer will adjust their optimal decisions to reduce carbon emissions. In terms of the manufacturer's total profit, an increase in carbon price will help increase the manufacturer's total profit, while an increase in carbon tax will reduce its total profit, which is similar to previous research, such as [13,14,18]. What differs from previous results is the impact on retailer's total profit; this study shows that it will first decrease and then increase as the carbon price increases but will increase with the increase in the carbon tax under the carbon cap-and-trade policy. In contrast, under the carbon tax policy, regardless of the carbon price or the carbon tax increases, the retailer's total profit will decrease.

In addition, to provide manufacturers and retailers with a decision-making reference in response to changes in the external environment, this study conducted a sensitivity analysis on the main parameters of the proposed model and obtained some meaningful management insights. For example, compared with previous studies that believe that an increase in demand rate can increase manufacturers' total profit [26,27], the findings of this study indicated that the total profit of the manufacturer will initially decrease and then increase as the demand rate rises when considering a supply chain system in which a manufacturer is the leader. Moreover, an increased manufacturer production rate does help the manufacturer reduce carbon emissions, but its total profit will also be reduced because the retailer reduces optimal order quantity. Finally, most previous related studies (see, for example, [13,14,18–20,30,35]) only presented the impact of changes in carbon emission parameters on the optimal solution, carbon emissions, and total profits. This study further compared the sensitivities of these parameters and obtained that the sensitivity of the manufacturer's carbon emissions generated by production activities is relatively high, while the sensitivity of the retailer's carbon emissions generated by purchasing activities is relatively high. That is to say, when low-carbonization strategies are necessary, the retailer or manufacturer that can effectively reduce the carbon emissions generated by these activities will significantly reduce entire carbon emissions.

Several future research directions can be considered. First of all, this model only considers the fixed market demand rate faced by the retailer. However, in reality, the demand for some commodities may be dependent on price, inventory level, low-carbon investment, etc.; thus, the variable demand rate can be taken into account in the future. Moreover, there may be multiple retailers or multiple channels in modern supply chain systems, so the proposed model can be extended to include production inventory problems involving multiple retailers or multiple channels. Furthermore, in this case, the algorithm developed in this study may not be applicable, so meta-heuristic algorithms can be considered to solve the model and compare the results [44]. This study can also take several practical scenarios, such as allowing shortages, trade credit, quantity discounts, or other carbon emission policies, into account.

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